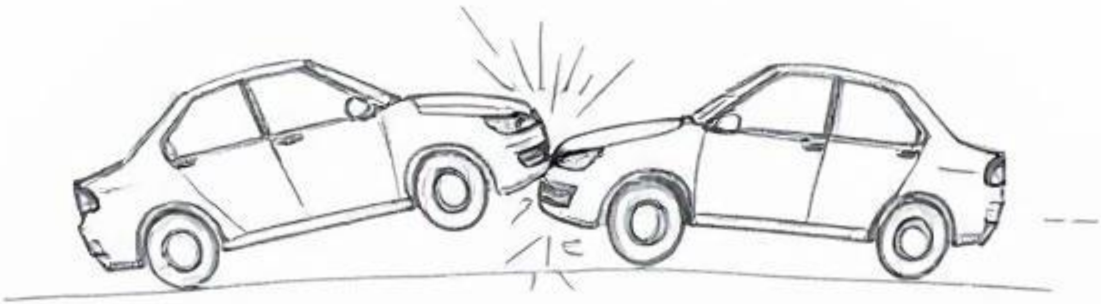


Collisions

V Vijaykiran
M.Sc., B.Ed., NET
SIGMA Physics Resource Centre
http://www.sigmaprc.in

Note : The notes given in this file is no substitute to the much detailed discussion held in the online/contact classes with active participation of students. It , at best, serves the purpose of ready reference for important concepts/derivations covered in the classes.

Collision

Collision is an interaction between two or more bodies leading to a rapid change in energy or linear momentum or both.

Collisions may be classified as

- **Elastic collisions**
  Collisions in which total linear momentum and total kinetic energy of the colliding bodies is conserved
  Examples : Collisions between the atoms of an ideal gas are assumed to be elastic

- **Inelastic collisions**
  Collisions in which only total linear momentum of the colliding bodies is conserved
  Examples : Collision of a ball with a bat, collision between particles of air

Final velocities (v_1 & v_2) in case of a 1-D elastic collision

Click here for simulation

Consider two bodies of masses m_1 and m_2. Let u_1 & u_2 be their initial velocities and v_1 & v_2 be their final velocities respectively

Using the law of conservation of linear momentum we get

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{(i)}$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \text{(ii)}$$

Using conservation of *KE* we get

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2)$$

Using equation (i) we get

$$(u_1 + v_1) = (v_2 + u_2)$$

$$u_1 - u_2 = v_2 - v_1 \quad \text{(iii)}$$

In a one dimensional elastic collision, relative velocity of approach is equal to relative velocity of separation.

Final velocities (v_1 & v_2) in case of a 1-D elastic collision

Click here for simulation

$$u_1 - u_2 = v_2 - v_1 \quad \text{iii}$$

$$v_2 = v_1 + u_1 - u_2$$

Substituting this in eq (i) we get

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2 (v_1 + u_1 - u_2)$$

$$v_1 (m_1 + m_2) = 2m_2u_2 + u_1 (m_1 - m_2)$$

$$\boxed{v_1 = \frac{2m_2u_2}{(m_1 + m_2)} + \frac{u_1 (m_1 - m_2)}{(m_1 + m_2)}}$$

Substituting the value of v_1 in eq (i) we get

$$\boxed{v_2 = \frac{2m_1u_1}{(m_1 + m_2)} + \frac{u_2 (m_2 - m_1)}{(m_1 + m_2)}}$$

Note : The above equations are applicable to one dimensional elastic collisions only.

Analysis of 1-D elastic collisions

Click here for simulation

$$v_1 = \frac{2m_2u_2}{(m_1+m_2)} + \frac{u_1(m_1-m_2)}{(m_1+m_2)}$$

$$v_2 = \frac{2m_1u_1}{(m_1+m_2)} + \frac{u_2(m_2-m_1)}{(m_1+m_2)}$$

Case I :

If $m_1 = m_2$ then

$$v_1 = \frac{2mu_2}{(m+m)} + \frac{u_1(m-m)}{(m+m)}$$

$$v_1 = u_2$$

$$v_2 = \frac{2mu_1}{(m+m)} + \frac{u_2(m-m)}{(m+m)}$$

$$v_2 = u_1$$

Velocities of the colliding bodies are exchanged in a one dimensional elastic collision between bodies of same mass.

Click here for simulation

Analysis of 1-D elastic collisions

$$v_1 = \frac{2m_2u_2}{(m_1+m_2)} + \frac{u_1(m_1-m_2)}{(m_1+m_2)}$$

$$v_2 = \frac{2m_1u_1}{(m_1+m_2)} + \frac{u_2(m_2-m_1)}{(m_1+m_2)}$$

Case II :

If $m_1 >> m_2$ and $u_2 = 0$

$$v_1 = \frac{0}{(m_1+m_2)} + \frac{u_1(m_1-m_2)}{(m_1+m_2)}$$

$$v_1 \approx u_1$$

$$v_2 = \frac{2m_1u_1}{(m_1+m_2)} + \frac{0(m_2-m_1)}{(m_1+m_2)}$$

$$v_2 \approx 2u_1$$

Heavy body continues to move with same velocity and the lighter body moves with twice the initial velocity of the heavier body

Analysis of 1-D elastic collisions

Click here for simulation

$$v_1 = \frac{2m_2u_2}{(m_1 + m_2)} + \frac{u_1(m_1 - m_2)}{(m_1 + m_2)}$$

$$v_2 = \frac{2m_1u_1}{(m_1 + m_2)} + \frac{u_2(m_2 - m_1)}{(m_1 + m_2)}$$

Case III :

If $m_1 << m_2$ and $u_2 = 0$

$$v_1 = \frac{0}{(m_1 + m_2)} + \frac{u_1(m_1 - m_2)}{(m_1 + m_2)}$$

$$v_1 \approx \frac{u_1(-m_2)}{(m_2)}$$

$$v_1 \approx -u_1$$

$$v_2 = \frac{2m_1u_1}{(m_1 + m_2)} + \frac{0(m_2 - m_1)}{(m_1 + m_2)}$$

$$v_2 \approx \frac{2m_1u_1}{(m_2)}$$

$$v_2 \approx 0$$

Lighter body rebounds with almost the same speed and the heavy body remains stationary

Completely inelastic collision

Click here for simulation

Using law of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{i}$$

$$m_1u_1 + m_2u_2 = (m_1 + m_2)\, v$$

$$\boxed{v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)}}$$

Coefficient of restitution

Coefficient of restitution is defined as the ratio of relative velocity of separation to relative velocity of approach

$$\boxed{e = \frac{(v_2 - v_1)}{(u_1 - u_2)}}$$

- Coefficient of restitution does not have any units
- $e = 1$ for perfectly elastic collisions
- $e = 0$ for perfectly inelastic collisions
- $0 < e < 1$ for partial elastic (or inelastic) collisions

Coefficient of restitution (in terms of heights)

Consider a body dropped from a height h_1. After collision let the body rebound to a height h_2. Coefficient of restitution is defined as

$$e = \frac{(v_2 - v_1)}{(u_1 - u_2)}$$

Let u_1 and v_1 represent initial and final velocities of the ground

$$e = \frac{(v_2)}{(-u_2)}$$

Using the relation $v^2 - u^2 = 2as$ for descent we get

$$u_2^{\,2} - 0 = 2gh_1$$

$$u_2 = \sqrt{2gh_1}$$

Using the relation $v^2 - u^2 = 2as$ for ascent we get

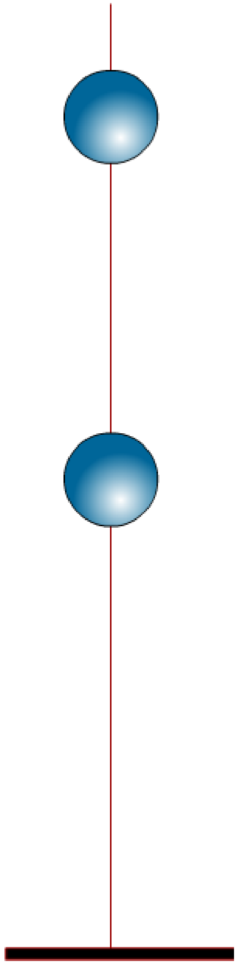
$$0 - v_2^{\,2} = -2gh_2$$

$$v_2 = \sqrt{2gh_2}$$

Substituting these in eq (i) we get

$$e = \frac{\sqrt{2gh_2}}{\sqrt{2gh_1}}$$

$$\boxed{e = \sqrt{\frac{h_2}{h_1}}}$$



Click here for simulation

Partially elastic (or inelastic) collisions

Click here for simulation

- Only linear momentum is conserved
- Kinetic energy is not conserved

$$v_1 = \frac{(e+1)\, m_2 u_2}{m_1 + m_2} + \frac{u_1 (m_1 - e m_2)}{m_1 + m_2}$$

$$v_2 = \frac{(e+1)\, m_1 u_1}{m_1 + m_2} + \frac{u_2 (m_2 - e m_1)}{m_1 + m_2}$$

These are general relation in case of 1-D elastic collisions.

Substituting e = 1 in the above relations gives v_1 and v_2 for perfectly elastic collisions

Substituting e = 0 in the above relations gives v_1 and v_2 for perfectly inelastic collisions